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A GENERALIZED CLASS OF ESTIMATORS FOR COMMON PARAMETERS OF TWO NORMAL DISTRIBUTIONS WITH KNOWN COEFFICIENT OF VARIATION

HOUSILA P. SINGH and N. P. KATYAR
J. N. K. Vishwa Vidyalaya, Jabalpur-482004

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SUMMARY

A general class of estimators for estimating common parameters θ^p ($p = 1, 2$) of two normal distributions has been suggested and its properties studied.

Keywords : Class of estimators; Common parameters; Normal distribution; Coefficient of variation.

Introduction

In various experiments, the coefficient of variation exhibits stability and its value may be fairly accurately known; see, e.g. Govindarajulu and Sahai [3], Sen [10, 11]. Utilizing the prior information on coefficient of variation 'C' several authors have discussed the problem of estimating the parameters θ and θ^2 of a normal distribution $N(\theta, \theta^2 C^2)$ and suggested a number of estimators. Very few authors have paid their attention towards the estimation of common parameters of two normal distributions utilizing the prior information on coefficient of variation. Reference may be made to the work done by Pandey and Singh [6] and Sahai *et al.* [8].

Let us consider two normal populations with common parameters say, θ and θ^2. Our investigations concern with the situations wherein the coefficients of variation are known for the two normal populations. Let $X_{11}, X_{12}, \ldots, X_{1n_1}$ and $X_{21}, X_{22}, \ldots, X_{2n_2}$ be random samples of sizes n_1

and n_2 respectively, from the populations $N(\theta, C_1^2\theta^2)$ and $N(\theta, C_2^2\theta^2)$ where C_i is the coefficient of variation of ith population. Further, let

$$\bar{X}_k = \sum_{j=1}^{n_k} (X_{kj}/n_k)$$

and

$$s_k^2 = (n_k - 1)^{-1} \sum_{j=1}^{n_k} (X_{kj} - \bar{X}_k)^2, \qquad k = 1, 2,$$

be the sample means and sample variances, respectively.

The conventional unbiased estimators of θ and θ^2 are, respectively, given by

$$d_1 = \frac{a_2 \bar{X}_1 + a_1 \bar{X}_1}{a_1 + a_2} \tag{1.1}$$

and

$$d_2 = \frac{(n_1 - 1)(s_1^2/C_1^2) + (n_2 - 1)(s_2^2/C_2^2)}{n_1 + n_2 - 2}, \tag{1.2}$$

where

$$a_k = \frac{C_k^2}{n_k}; \qquad k = 1, 2$$

The variances of d_1 and d_2 are given by

$$V(d_1) = \frac{\theta^2 a_1 a_2}{a_1 + a_2} \tag{1.3}$$

and

$$V(d_2) = \frac{2\,\theta^4}{(n_1 + n_2 - 2)} \tag{1.4}$$

Improvements in the estimators can be made if we are prepared to sacrifice unbiasedness. One such estimator was first proposed by Searls [9] assuming the coefficient of variation to be known for estimating θ. Following the same approach as adopted by Searls [9], Pandey and Singh [6] suggested a class of estimators for θ as

$$d_3 = \lambda \left(\frac{a_2 \bar{x}_1 + a_1 \bar{x}_2}{a_1 + a_2} \right) \tag{1.5}$$

where λ is a suitably chosen constant to be determined such that mean squared error of d_3 is minimum. The optimum value of λ and minimum mean squared error of d_3 obtained by Pandey and Singh [6] are, respectively, given by

$$\lambda_{opt} = \frac{(a_1 + a_2)}{a_1 + a_2 + a_1 a_2} \tag{1.6}$$

and

$$\min \cdot \text{MSE}\,(d_3) = \frac{a_1\, a_2\, \theta^2}{(a_1 + a_2 + a_1 a_2)} \tag{1.7}$$

One can also define a class of estimators for θ as

$$d_4 = \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 \tag{1.8}$$

where λ_1 and λ_2 are suitably chosen constants to be determined such that MSE of d_4 is minimum.

It can be easily shown that the minimum MSE of d_4 is same as of d_3 in (1.7) for the optimum values of λ_1 and λ_2, which are given as follows:

$$\lambda_{10} = \frac{a_2}{a_1 + a_2 + a_1\, a_2} \quad \text{and} \quad \lambda_{20} = \frac{a_1}{a_1 + a_2 + a_1\, a_2} \tag{1.9}$$

Sahai, Prasad and Rani [8] have suggested a wide class of estimators for θ as

$$d_5 = \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2 + \lambda_3 s_1 + \lambda_4 s_4 \tag{1.10}$$

where λ_i's ($i = 1$ to 4) are suitably chosen constants to be determined such that MSE of d_5 is minimum. They have shown that the minimum MSE of d_5 is smaller than that of d_3 and d_4.

In the situations, where the coefficient of variation is known, the estimation of variance reduces to the estimation of the square of the population mean, θ, see Govindarajulu and Sahai [3] and Das [1].

In this paper we have proposed a very general class of estimators for θ^p ($p = 1, 2$) exploiting the *apriori* information in terms of coefficients of variation C_1 and C_2. Exact expressions for bias and mean squared errors (MSE) of the proposed class of estimators have been derived. The optimum estimator in the class is also identified.

2. Proposed Class of Estimators

We propose the class of estimators of the common parameters θ^p ($p = 1, 2$) as the linear function:

$$d_n = \left[\sum_{i=0}^{m} W_{1i}\, \bar{x}_1^i\, s_1^{-i+p} + \sum_{i=0}^{m} W_{2i}\, \bar{x}_2^i\, s_2^{-i+p} \right] = \sum_{i=0}^{m} \sum_{k=1}^{2} W_{ki}\, \bar{x}_k^i\, s_k^{-} +$$

$$= \sum_{k=1}^{2} \underset{\sim}{S_k}\, \underset{\sim}{W_k} \tag{2.1}$$

where

$$\underset{\sim}{S}_k = [s_k^p,\ \bar{x}_k\, s_k^{p-1},\ \bar{x}_k^2\, s^{p-2},\ \ldots,\ \bar{x}_k^{m-1}\ s_k^{p-m+1},\ \bar{x}_k^m\ s_k^{p-m}]_{1\times(m+1)}$$

$$\underset{\sim}{W}'_k = [W_{k0},\ W_{k1},\ W_{k2},\ \ldots,\ W_{k(m-1)},\ W_{km}]_{1\times(m+1)}$$

and W_{ki}'s; $i = 0$ to m; $k = 1, 2$ are suitably chosen constants to be determined such that MSE of d_n is minimum, m being a positive integer.

To evaluate the bias and MSE of d_n we shall use the following results : As the parent populations are normal, then

$$\frac{(n_k - 1)\, s_k^2}{\theta^2\, C_k^2} \sim \chi^2_{(n_k-1)}, \qquad k = 1, 2$$

and

$$\bar{x}_k \sim N(\theta,\ a_k\, \theta^2) \quad \text{independently of } s_k^2, \text{ where } a_k = \frac{C_k^2}{n_k}, \qquad k = 1, 2$$

hence

$$E(s_k^j) = \left(\frac{2\, n_k}{n_k - 1}\right)^{j/2} \frac{\sqrt{\left(\frac{n_k + j - 1}{2}\right)}}{\sqrt{\left(\frac{n_k - 1}{2}\right)}} \cdot (a_k\, \theta^2)^{j/2} = I_{(k)}^{(j)}\, \theta^j, \qquad (2.2)$$

$$j = \pm 1,\ \pm 2, \ldots, \pm m$$
$$k = 1, 2$$

and

$$E(\bar{x}_k^q) = \sum_{i=0}^{r=[q/2]} 2^i \binom{q}{2i} \left(\frac{\sqrt{\frac{2i+1}{2}}}{\sqrt{\frac{1}{2}}}\right) (a_k)^i\, \theta^q, \qquad (2.3)$$

$$= \theta^q\, \delta_{(k)}^{(q)}, \quad q \text{ being a positive integer}$$

where

$$I_{(k)}^{(j)} = \left(\frac{2\, n_k\, a_k}{n_k - 1}\right)^{j/2} \frac{\sqrt{\frac{n_k + j - 1}{2}}}{\sqrt{\frac{n_k - 1}{2}}}$$

and

$$\delta_{(k)}^{(q)} = \sum_{i=0}^{r=[q/2]} 2^i \binom{q}{2i} \left(\sqrt{\frac{2i+1}{2}} \Big/ \sqrt{\frac{1}{2}}\right) (a_k)^i$$

The bias of d_h is given by

$$B(d_h) = -\theta^p \left[1 - \sum_{k=1}^{2} \underset{\sim}{D_k} \underset{\sim}{W_k} \right] \tag{2.4}$$

where

$$\frac{E(S_k)}{\theta^p} = D_k = [I_{(k)}^{(p)}, \delta_{(k)}^{(1)} I_{(k)}^{(p-1)}, \delta_{(k)}^{(2)} I_{(k)}^{(p-2)}, \ldots, \delta_{(k)}^{(m)} I_{(k)}^{(p-m)}]$$

The mean squared error of d_h is given by

$$\text{MSE}(d_h) = \theta^{2p} \left[\sum_{k=1}^{2} (\underset{\sim}{W_k'} \underset{\sim}{\Sigma_k} \underset{\sim}{W_k} - 2 \underset{\sim}{D_k} \underset{\sim}{W_k}) + 2 \underset{\sim}{W_1'} \underset{\sim}{\Sigma_{12}} \underset{\sim}{W_2} + 1 \right] \tag{2.5}$$

where

$$\sum_{12} = \frac{E(\underset{\sim}{S_1'} \underset{\sim}{S_2})}{\theta^{2p}} \quad \text{and} \sum_k = \frac{E(\underset{\sim}{S_k'} \underset{\sim}{S_k})}{\theta^{2p}}; \quad k = 1, 2$$

are defined elsewhere. Differentiating (2.5) with respect to $\underset{\sim}{W_k'}$, $k = 1, 2$ partially are equating them to zero, we obtain the following normal equation :

$$\begin{bmatrix} \underset{\sim}{\Sigma_1} & \underset{\sim}{\Sigma_{12}} \\ \underset{\sim}{\Sigma_{12}} & \underset{\sim}{\Sigma_2} \end{bmatrix} \begin{bmatrix} \underset{\sim}{W_1} \\ \underset{\sim}{W_2} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{D_1'} \\ \underset{\sim}{D_2'} \end{bmatrix} \tag{2.6}$$

Using Cramer's rule of solving the simultaneous equation we obtain the optimum values of W_1 and W_2 as

$$\left. \begin{aligned} \underset{\sim}{W_1} &= \underset{\sim}{\Delta}^{-1} \underset{\sim}{\Delta_1} = \underset{\sim}{W_1^*}, \quad \text{(say)} \\ \underset{\sim}{W_2} &= \Delta^{-1} \underset{\sim}{\Delta_2} = \underset{\sim}{W_2^*}, \quad \text{(say)} \end{aligned} \right\} \tag{2.7}$$

where

$$\underset{\sim}{\Delta} = (\underset{\sim}{\Sigma_1} \underset{\sim}{\Sigma_2} - \underset{\sim}{\Sigma_{12}'} \underset{\sim}{\Sigma_{12}})$$

$$\underset{\sim}{\Delta_1} = (\underset{\sim}{\Sigma_2} \underset{\sim}{D_1'} - \underset{\sim}{\Sigma_{12}} \underset{\sim}{D_2'})$$

$$\underset{\sim}{\Delta_2} = (\underset{\sim}{\Sigma_1} \underset{\sim}{D_2'} - \underset{\sim}{\Sigma_{12}} \underset{\sim}{D_1'})$$

Hence the minimum MSE of d_h is given by

$$\min \cdot \text{MSE}(d_h) = \theta^{2p} \left[1 - \sum_{k=1}^{2} \underset{\sim}{D_k} \underset{\sim}{\Delta}^{-1} \underset{\sim}{\Delta_k} \right], \quad k = 1, 2 \tag{2.8}$$

substituting the optimum values of $\underset{\sim}{W_k}$ from (2.7) in (2.4), we obtain the bias of d_h as

$$B^*(d_h) = -\frac{\min \cdot \text{MSE}(d_h)}{\theta^p} \tag{2.9}$$

$$\frac{E(\underset{\sim}{S}_k\, \underset{\sim}{S}_k)}{\theta^{2p}} = \underset{\sim}{\Sigma}_k = \begin{bmatrix} I^{(2p)}_{(k)} & , & \delta^{(1)}_{(k)} I^{(2p-1)}_{(k)} & , \ldots, & \delta^{(m)}_{(k)} I^{(2p-m)}_{(k)} \\ \delta^{(1)}_{(k)} I^{(2p-1)}_{(k)} & , & \delta^{(1)}_{(k)} I^{(2p-1)}_{(k)} & , \ldots, & \delta^{(m+1)}_{(k)} I^{(2p-m-1)}_{(k)} \\ \delta^{(2)}_{(k)} I^{2(p-1)}_{(k)} & , & \delta^{(3)}_{(k)} I^{(2p-3)}_{(k)} & , \ldots, & \delta^{(m+2)}_{(k)} I^{(2p-m-2)}_{(k)} \\ \vdots & & \vdots & & \vdots \\ \delta_{(m-1)}^{(k)} I^{(2p-m+1)}_{(k)} & , & \delta^{(m)}_{(k)} I^{(2p-m)}_{(k)} & , \ldots, & \delta^{(2m-1)}_{(k)} I^{(2p-2m+1)}_{(k)} \\ \delta^{(m)}_{(k)} I^{(2p-m)}_{(k)} & , & \delta^{(m+1)}_{(k)} I^{(2p-m-1)}_{(k)} & , \ldots, & \delta^{(2m)}_{(k)} I^{(2p-m)}_{(k)} \end{bmatrix}$$

$$\frac{E(\underset{\sim}{S}'_1\, \underset{\sim}{S}_2)}{\theta^{2p}} = \underset{\sim}{\Sigma}_{12} = \begin{bmatrix} I^{(p)}_{(1)} I^{(p)}_{(2)} & , & \delta^{(1)}_{(2)} I^{(p)}_{(1)} I^{(p-1)}_{(2)} & , \ldots, & \delta^{(m)}_{(2)} I^{(p)}_{(1)} I^{(p-2)}_{(2)} \\ \delta^{(1)}_{(1)} I^{(p-1)}_{(1)} I^{(p)}_{(2)} & , & \delta^{(1)}_{(1)} \delta^{(1)}_{(2)} I^{(p-1)}_{(1)} I^{(p-1)}_{(2)} & , \ldots, & \delta^{(1)}_{(1)} \delta^{(m)}_{(2)} I^{(p-1)}_{(1)} I^{(p-m)}_{(2)} \\ \delta^{(2)}_{(1)} I^{(p)}_{(2)} I^{(p-2)}_{(1)} & , & \delta^{(2)}_{(1)} \delta^{(1)}_{(2)} I^{(p-2)}_{(1)} I^{(p-1)}_{(2)} & , \ldots, & \delta^{(2)}_{(1)} \delta^{(m)}_{(2)} I^{(p-2)}_{(1)} I^{(p-m)}_{(2)} \\ \vdots & & \vdots & & \\ \delta^{(m-1)}_{(1)} I^{(p-m+1)}_{(1)} I^{(p)}_{(2)} & , & \delta^{(m-1)}_{(1)} \delta^{(1)}_{(2)} I^{(p-m+1)}_{(1)} I^{(p-1)}_{(2)} & , \ldots, & \delta^{(m-1)}_{(1)} \delta^{(m)}_{(2)} I^{(p-m+1)}_{(1)} I^{(p-m)}_{(2)} \\ \delta^{(m)}_{(1)} I^{(p-m)}_{(1)} I^{(p)}_{(2)} & , & \delta^{(m)}_{(1)} \delta^{(1)}_{(2)} I^{(p-m)}_{(1)} I^{(p-1)}_{(2)} & , \ldots, & \delta^{(m)}_{(1)} \delta^{(m)}_{(2)} I^{(p-m)}_{(1)} I^{(p-m)}_{(2)} \end{bmatrix}$$

Apparently the minimum MSE of d_h in (2.8) is less than that of d_3 and d_5 considered by Pandey and Singh [6] and Sahai, Prasad and Rani [8], respectively as d_h is a class bigger than d_3, d_4 and d_5. It is to be pointed out that the minimum MSE of d_h would always be smaller than that of any subclass of d_h. Beauty of this generalized estimator is that one can immediately obtain the expressions for biases and MSES of the estimators of θ and θ^2.

Further, if one is interested in estimating the parameters θ^p $(p = 1, 2)$ of a normal distribution having mean θ and variance $\theta^2 C^2$, i.e. $N(\theta, \theta^2 C^2)$, then the estimator d_h defined in (2.1) reduces to :

$$d_h^* \sum_{i=0}^{m} W_i \bar{x}^i s^{-i+p} = \underset{\sim}{S} \underset{\sim}{W} \tag{2.10}$$

where

$$\bar{x} = n^{-1} \sum_{i=1}^{n} x_i, \quad s^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})^2,$$

$$\underset{\sim}{S} = [s^p, \bar{x} s^{p-1}, \ldots, \bar{x}^{m-1} s^{p-m+1}, \bar{x}^m s^{p-m}]_{1\times(m+1)}$$

and

$$\underset{\sim}{W}^1 = [W_0, W_1, W_2, \ldots, W_{m-1}, W_m]_{1\times(m+1)}.$$

The bias and MSE of d_h^* can be obtaned from (2.4) and (2.5) respectively and are given as follows :

$$B(d_h^*) = -\theta^p (1 - \underset{\sim}{D} \underset{\sim}{W}) \tag{2.11}$$

and

$$\text{MSE}(d_h^*) = \theta^{2p} [\underset{\sim}{W}^1 \underset{\sim}{\Sigma} \underset{\sim}{W} - 2 \underset{\sim}{D} \underset{\sim}{W} + 1], \tag{2.12}$$

where

$$\underset{\sim}{D} = [I^{(p)}, \delta^{(1)} I^{(p-1)}, \delta^{(2)} I^{(p-2)}, \ldots, \delta^{(m)} I^{(p-m)}]$$

and

$$\begin{bmatrix} I^{(2p)}, \delta^{(1)} I^{(2p-1)} & , \ldots, & \delta^{(m)} I^{(2p-m)} \\ \delta^{(1)} I^{(2p-1)}, \delta^{(2)} I^{2(p-1)} & , \ldots, & \delta^{(m+1)} I^{(2p-m-1)} \\ \vdots & & \vdots \\ \delta^{(m-1)} I^{(2p-m+1)} & , \ldots, & \delta^{(2m-1)} I^{(2p-m+1)} \\ \delta^{(m)} I^{(2p-m)} & , \ldots, & \delta^{(2m)} I^{2(p-m)} \end{bmatrix}_{(m+1)\times(m+1)}$$

The MSE of d_h^* is minimized for

$$\underset{\sim}{W} = \underset{\sim}{\Sigma}^{-1} \underset{\sim}{D}' \tag{2.13}$$

Hence the resulting bias and minimum MSE of d_h^* are given by

$$B^*(d_h) = -\frac{\min \cdot \text{MSE}\,(d^*)}{\theta^p} \tag{2.14}$$

and

$$\min \cdot \text{MSE}\,(d_h^*) = \theta^{2p}\,(1 - \underset{\sim}{D}\,\underset{\sim}{\Sigma}^{-1}\,\underset{\sim}{D}') \tag{2.15}$$

Estimators, known from the literature, listed in Tables 1 and 2 can be identified as special cases of d_h^*.

TABLE 1—KNOWN ESTIMATORS FOR θ

Author(s) Name	*Choices of weights*				*Estimator*
	w_0	w_1	w_2	w_3	
Searls [9]	0	w	0	0	$e_1 = w\bar{x}$
Khan [5]	$w/I^{(1)}$	$1 - w$	0	0	$e_2 = w(s/I^{(1)}) + (1-w)\bar{x}$
Sen [10, 11]	$(1-w)\,C^{-1}$	w	0	0	$e_3 = (1-w)\frac{s}{C} + w\bar{x}$
Govindarajulu and Sahai [3], Gleser and Healy [2] and Upadhyaya and Singh [21]	w_0	w_1	0	0	$e_4 = w_0\, s + w_1\, \bar{x}$
Prasad and Sahai [7]	w_0	w_1	0	w_3	$e_5 = w_0 s + w_1 x + w_3 \frac{\bar{x}^3}{s^2}$
Conventional unbiased estimator	0	1	0	0	$_6 = \bar{x}$
Another unbiased estimator	$\frac{1}{I^{(1)}}$	0	0	0	$e_7 = \frac{s}{I^{(1)}}$

TABLE 2—KNOWN ESTIMATORS FOR θ^2

Author(s) Name	*Choices of weight*					*Estimator*
	w_0	w_1	w_2	w_3	w_4	
Govindarajulu and Sahai [3]	w_0	0	w_2	0	0	$t_1 = w_0 s^2 + w_2 \overline{x}^2$
Das [1]	0	0	$(1+a)^{-1}$	0	0	$t_2 = \frac{\overline{x}^2}{(1+a)},\ a = \frac{c^2}{n}$
Srivastava, Dwivedi and Bhatnagar [18]	$-\frac{1}{n}\left(\frac{n-1}{n+1}\right)$	0	1	0	0	$t_3 = \overline{x}^2 - \frac{1}{n}\left(\frac{n-1}{n+1}\right) s^2$
Singh and Upadhyaya [15]	w_0	0	w_2	0	w_4	$t_4 = w_0 s^2 + w_2 \overline{x}^2 + w_4 \frac{\overline{x}^4}{s^2}$
Singh [16]	w_0	0	w_2	w_3	0	$t_5 = w_0 s^2 + w_2 \overline{x}^2 + w_3 \frac{\overline{x}^3}{s}$
Singh [14]	w_0	w_1	w_2	0	0	$t_6 = w_0 s^2 + w_1 \overline{x} s + w_2 \overline{x}^2$
Singh, Pandey and Hirano [12]	w_0	0	0	0	0	$t_7 = w_0 s^2$
An estimator	0	0	1	0	0	$t_8 = \overline{x}^2$
Unbiased estimator	$c^{\overline{2}}$	0	0	0	0	$t_9 = \frac{s^2}{C^2}$
Singh [13]	0	0	w_2	0	0	$t_{10} = w_2 \overline{x}^2$

The estimators cited in Tables 1 and 2 are subclasses of the generalized estimator d_h^*. Hence, their biases and MSEs can be easily obtained from the expressions (2.11) and (2.12), respectively. It is also interesting to note that the minimum MSE of d_h^* is smaller than that of the estimators cited in Tables 1 and 2 and other subclasses of d_h^* as they are the particular cases of the generalized class of estimators d_h^*.

3. Numerical Illustrations

In this section we have illustrated the performance of various estimators of (a) the common parameter θ in two normal distributions $N(\theta, C_1^2\theta^2)$ and $N(\theta, C_2^2\theta^2)$; and (b) the parameter θ in single normal population $N(\theta, C^2\theta^2)$.

(a) We consider the following estimators of the common parameter θ which are essentially particular members of the proposed generalized class of estimators d_h defined in (2.1) :-

(i) $T_1 = w_1\bar{x}_1 + w_2\bar{x}_2$

Which has same minimum mean square error as that of Pandey and Singh [6] estimator.

$$d_3 = \lambda\,\frac{(a_2\bar{x}_1 + a_1\bar{x}_2)}{(a_1 + a_2)}, \qquad \lambda \text{ being a constant.}$$

(ii) $T_2 = w_1\bar{x}_1 + w_2\bar{x}_2 + w_3s_1 + w_4s_2$

reported by Sahai *et al.* (1983).

(iii) $T_3 = w_1\bar{x}_1 + w_2\bar{x}_2 + w_3s_1 + w_4s_2 + w_5(\bar{x}_1^3/s_1^2) + w_6(\bar{x}_2^3/s_2^2)$

where w_i's, $i = 1$ to 6 are suitably chosen constants.

In order to examine the behaviour of these estimators, we have computed the relative efficiency of T_2 and T_3 with respect to T_1 (or d_3) and compiled in Tables 3 to 9 for different values of n_1, n_2, c_1 and c_2. The relative efficiency of an estimator T_i, $i = 2, 3$ with respect to T_1 (or d_3) is defined by the formula

$$RE(T_i, T_1) = \frac{\min\cdot \text{MSE}(T_1)}{\min\cdot \text{MSE}(T_i)}; \qquad i = 2, 3$$

Further, we denote

$$E_1 = RE(T_2, T_1) \quad \text{and} \quad E_2 = RE(T_3, T_1).$$

The min · MSE (T_j); $j = 1, 2, 3$ (the minimum mean squared error of T_j; $j = 1, 2, 3$) can be obtained from (2.8).

TABLE 3—RELATIVE EFFICIENCIES OF T_2 AND T_3 WITH RESPECT TO T_1 (in percent) FOR $n_1 = 5$; $n_2 = 10$

c_1 ↓ \ ca →		0.1	0.25	0.5	1.0	2.0
0.1	E_1	101.725	103.919	104.788	105.069	105.144
	E_2	101.935	103.989	104.885	105.269	105.285
0.25	E_1	102.395	110.748	121.407	128.464	131.021
	E_2	103.001	110.999	122.005	129.100	132.352
0.5	E_1	100.536	114.311	142.463	183.555	210.222
	E_2	102.895	115.463	143.470	185.626	213.275
1.0	E_1	102.575	115.604	156.309	261.889	404.732
	E_2	103.626	117.441	158.767	265.130	409.082
2.0	E_1	102.584	115.964	161.307	311.446	645.309
	E_2	104.752	121.286	181.930	385.067	839.724

TABLE 4—RELATIVE EFFICIENCIES OF T_2 AND T_3 WITH RESPECT TO T_1 (in percent) FOR $n_1 = 5$ AND $n_2 = 15$

↓ c_2 \ c_1 →		0.1	0.25	0.5	1.0	2.0
0.1	E_1	101.645	104.443	105.868	106.380	106.522
	E_2	101.881	104.631	106.434	107.042	107.221
0.25	E_1	102.083	110 256	123.348	134.292	138.844
	E_2	102.783	110.963	125.432	137.672	142.911
0.5	E_1	102.165	112.613	140.642	191.445	233.011
	E_2	103.001	113.643	143.817	199.485	245.539
1.0	E_1	102.187	113.382	149.876	256.762	437.642
	E_2	103.551	114.385	150.356	259.819	446.346
2.0	E_1	102.192	113.589	152.884	290.841	648.669
	E_2	103.653	118.642	172.950	365.170	865.803

TABLE 5—RELATIVE EFFICIENCIES OF T_2 AND T_3 WITH RESPECT TO T_1 (in percent) FOR $n_1 = 5$, $n_2 = 25$

c_2 ↓ \ c_1 →		0.1	0.25	0.5	1.0	2.0
0.1	E_1	101.642	105.471	108.202	109.371	109.717
	E_2	101.992	106.771	109.802	110.937	111.315
0.25	E_1	101.907	110.256	127.233	156.501	156.494
	E_2	102.102	111.497	131.532	158.051	165.687
0.5	E_1	101.952	111.706	140.737	207.153	280.883
	E_2	102.001	113.600	156.899	223.643	309.055
1.0	E_1	101.952	112.141	146.501	259.002	502.373
	E_2	102.762	112.724	151.625	276.212	546.641
2.0	E_1	101.962	112.254	148.206	280.883	679.890
	E_2	103.332	116.666	165.170	345.151	886.979

TABLE 6—RELATIVE EFFICIENCIES OF T_2 AND T_3 WITH RESPECT TO T_1 (in percent) FOR $n_1 = 10$, $n_2 = 5$

c_2 ↓ \ c_1 →		0.1	0.25	0.5	1.0	2.0
0.1	E_1	101.726	102.396	102.537	102.575	102.984
	E_2	101.801	102.931	102.991	102.998	103.050
0.25	E_1	103.809	110.748	114.311	115.604	115.964
	E_2	103.919	110.891	114.912	116.020	121.244
0.5	E_1	104.784	121.407	142.463	156.630	161.307
	E_2	104.991	122.310	143.011	157.769	181.926
1.0	E_1	105.069	128.468	183.555	261.889	311.446
	E_2	105.089	129.841	184.005	281.923	385.065
2.0	E_1	105.144	131.021	210.222	404.732	645.309
	E_2	105.424	132.331	211.983	759.330	839.724

TABLE 7—RELATIVE EFFICIENCIES OF T_2 AND T_3 WITH RESPECT TO T_1 (in percent) FOR $n_1 = 10$, $n_2 = 15$

↓ c_2 \ c_1 →		0.1	0.25	0.5	1.0	2.0
0.1	E_1	101.723	104.309	105.438	105.884	105.993
	E_2	101.825	104.699	105.841	106.002	106.831
0.25	E_1	102.267	110.761	123.129	132.456	136.095
	E_2	103.424	111.525	124.290	134.051	138.476
0.5	E_1	102.375	113.689	142.814	191.467	227.763
	E_2	103.543	116.526	146.065	193.533	235.141
1.0	E_1	102.402	114.688	154.486	267.689	449.959
	E_2	103.681	118.742	165.977	281.351	491.073
2.0	E_1	102.410	114.961	158.326	311.818	719.159
	E_2	103.510	119.414	174.058	376.520	756.202

TABLE 8—RELATIVE EFFICIENCIES OF T_2 AND T_3 WITH RESPECT TO T_1 (in percent) FOR $n_1 = 15$, $n_2 = 20$

↓ c_2 \ c_1 →		0.1	0.25	0.5	1.0	2.0
0.1	E_1	101.928	103.546	104.271	104.539	104.484
	E_2	102.494	103.946	104.875	104.991	105.002
0.25	E_1	103.012	112.334	121.029	125.865	127.441
	E_2	103.981	112.899	122.031	126.105	129.142
0.5	E_1	103.277	114.718	147.875	183.337	202.278
	E_2	104.005	117.807	149.321	185.004	203.367
1.0	E_1	103.350	116.199	164.330	287.509	421.444
	E_2	104.512	118.547	169.221	289.747	431.168
2.0	E_1	103.368	116.649	170.316	372.741	792.341
	E_2	104.992	119.105	178.849	395.132	799.713

TABLE 9—RELATIVE EFFICIENCIES OF T_2 AND T_3 WITH RESPECT TO T_1 (in percent) FOR $n_1 = 15$, $n_2 = 25$

$c_1 \rightarrow$ / $\downarrow c_2$		0.1	0.25	0.5	1.0	2.0
0.1	E_1	101.683	103.549	104.209	104.417	104.472
	E_2	102.047	104.167	104.836	105.213	105.344
0.25	E_1	102.457	110.513	119.769	125.333	127.254
	E_2	103.001	112.167	122.984	129.545	131.812
0.5	E_1	102.630	114.617	141.854	178.356	200.202
	E_2	103.350	115.607	148.809	191.563	217.179
1.0	E_1	102.677	116.199	158.091	264.356	402.858
	E_2	103.450	119.025	168.284	292.848	455.610
2.0	E_1	102.689	116.649	164.330	326.507	712.600
	E_2	103.651	120.089	174.956	435.103	821.196

It is observed from Tables 3 to 9 that the relative efficiency of T_3 with respect to T_1 is larger than that of T_2 with respect to T_1. Hence, the class of estimators T_3 is more efficient than T_2 and T_1 (or d_3) both. Thus, it leads to the conclusion that minimum MSE estimator obtained from a bigger class is superior to those minimum MSE estimator obtained from smaller class. It also follows from the tables that the gain in efficiency is quite significant for larger values of c_1 and c_2. However, the gain decreases with an increase in sample size n_1 and/or n_2. Nevertheless, the gain is rather substantial and is worth going for.

(b) Here we consider the estimators of the parameter θ in single normal population $N(\theta, c^2\theta^2)$ as—

(i) $d_1^* = w_0 s + w_1 \bar{x}$

(ii) $d_2^* = w_0 s + w_1 \bar{x} + w_2 (\bar{x}^3/s^2)$;

where, w_i, $i = 0, 1, 2$ is suitably chosen constant.

It is to be noted that the estimators d_1^* and d_2^* are particular cases of the generalized class of estimators d_h^* defined in (2.10).

In Table 10 we give the values of relative efficiencies of d_1^* and d_2^* with respect to sample mean $\bar{x}$ for various values of sample sizes $n = 10$, 20 and 50; and of $c^2 = 0.1$, 0 2, 0.5, 1.0, 2.0 (see Prasad and Sahai [7]). The relative efficiency of d_i^*; $i = 1, 2$ with respect to sample mean $\bar{x}$ is defined by

$$E_i^* = RE\,(d_i^*, \bar{x}) = \frac{\text{MSE}\,(\bar{x})}{\text{min}\,.\,\text{MSE}\,(d_i^*)}\,;\, i = 1, 2$$

TABLE 10— RELATIVE EFFICIENCIES OF d_1^* and d_2^* WITH RESPECT TO SAMPLE MEAN $\bar{x}$ (in Percent)

↓ c^2	n →	Sample size 10	20	50
0.1	E_1^*	118.541	111.524	119.702
	E_2^*	118.982	119.260	119.983
0.5	E_1^*	137.082	133.181	139.403
	E_2^*	138.012	138.520	140.171
1.0	E_1^*	285.412	292.598	297.015
	E_2^*	293.371	298.185	301.997
2.0	E_1^*	470.825	485.197	494.031
	E_2^*	489.515	498.269	504.859
5.0	E_1^*	1027.062	1062.991	1085.077
	E_2^*	1076.794	1098.659	1119.729
10.0	E_1^*	1954.124	2025.983	2070.154
	E_2^*	2045.778	2095.559	2141.528

Table 10 exhibits that for relatively larger samples, we may not have practically significant gain in efficiency unless c is rather big. For example, for $c^2 = 10$ (the C.V. $c = 3.16278$); even for sample as large as $n = 50$, the gain is worth going for. It also follows from Table 10 that the minimum MSE estimator obtained from a bigger class is more precise than that obtained from a smaller class.

4. When Weights are Unknown

In practice, the coefficients of variation C_1 and C_2 are rarely known so that the estimator d_h is of no practical utility. However, one may have a figure fairly close to the true values of the coefficients of variation from one's long association with the experimental material or from other experimental investigations or from some extraneous source; see Srivastava [17]. Often, the coefficient of variation may exhibit a stability in repeated experiments and it may be possible to make a reasonable guess. On the other hand, if the coefficients of variation are not known, these can be estimated from a large sample size n' ($\geqslant n$) and the estimated values are substituted in the optimum weights w_i^*, $i = 1$ to p in (2.7), say $\hat{w}_i^*$; $i = 1$ to p, the MSE's of estimated weights $\hat{w}_i^*$; $i = 1$ to p relative to that of w_i^*; $i = 1$ to p will be larger by an insignificant amount leading to near optimality; see Sen [10]. An elegant approach has also been described by Tripathi *et al.* [20] when the weights are approximately known.

The guessed (or estimated) values and the true values of coefficients of variation may be expressed as

$$\hat{c}_1 = \alpha\, C_1 \text{ and } \hat{c}_2 = \alpha\, C_2$$

where C_1 and C_2 are the true values of the coefficients of variation and α is any positive constant. Thus, the proposed estimator will be

$$\hat{d}_h = \sum_{k=1}^{2} \underset{\sim}{S_k}\, \underset{\sim}{\hat{W}_k} \tag{4.1}$$

where,

$$\underset{\sim}{S_k} = \left[\, s_k^p,\ \bar{x}_k\, s_k^{p-1},\ \bar{x}_k^2\, s_k^{p-2},\ \ldots,\ \bar{x}_k^{m-1}\, s_k^{p-m+1},\ \bar{x}_k^m\, s_k^{p-m} \right]$$

$$\underset{\sim}{W}'_k = [\hat{w}_{k0},\ \hat{w}_{k1},\ \hat{w}_{k2},\ \ldots,\ \hat{w}_{k\,(m-1)},\ \hat{w}_{km}]_{1\times(m+1)}$$

and $\hat{w}_{ki}$'s, $i = 0$ to m; $k = 1, 2$ are the guessed (or estimated) values of optimum weights w_{ki}'s, $i = 0$ to m; $k = 1, 2$ in (2.7) obtained after inserting $\hat{c}_1 = \alpha\, C_1$ and $\hat{c}_2 = \alpha\, C_2$ ($\alpha > 0$) in w_{ki}'s in (2.7). Such estimation technique has been used by various authors including Hirono [4], Pandey and Singh [6] and Srivastava *et al.* [19].

The bias and MSE of $\hat{d}_h$ are, respectively, given by

$$B(\hat{d}_h) = -\theta^p \left[1 - \sum_{k=1}^{2} \underset{\sim}{D}_k \hat{\underset{\sim}{W}}_k\right] \tag{4.2}$$

and

$$\text{MSE}(\hat{d}_h) = \theta^{2p}\left[\sum_{k=1}^{2} (\hat{\underset{\sim}{W}}_k' \underset{\sim}{\Sigma}_k \hat{\underset{\sim}{W}}_k - 2 D_k \hat{\underset{\sim}{W}}_k) + 2 \hat{\underset{\sim}{W}}_1' \Sigma_{12} \hat{\underset{\sim}{W}}_2 + 1\right] \tag{4.3}$$

where $\underset{\sim}{D}_k$, $\underset{\sim}{\Sigma}_k$, $k = 1, 2$ and $\underset{\sim}{\Sigma}_{12}$ are same as defined in Section 2.

Let d_p; $p = 1, 2$, defined in (1.1) and (1.2), be the conventional unbiased estimators of θ^p with variances $V(d_p)$; $p = 1, 2$ given in (1.3) and (1.4), respectively. In order to have MSE $(\hat{d}_h) \leqslant V(d_p)$, we should have the following inequality :

$$\theta^{2p}\left[\sum_{k=1}^{1} (\hat{\underset{\sim}{W}}_k' \underset{\sim}{\Sigma}_k \hat{\underset{\sim}{W}}_k - 2 \underset{\sim}{D}_k \hat{\underset{\sim}{W}}_k) + 2 \hat{W}_1' \underset{\sim}{\Sigma}_{12} \underset{\sim}{W}_2 + 1\right] - V(d_p) \leqslant 0. \tag{4.4}$$

It is to be noted that the weights $\underset{\sim}{W}_k$; $k = 1, 2$ involve a positive constant α. Therefore, from the inequality (4.4) the ranges can be computed for the given values of C_1, C_2, n_1 and n_2 under which the performance of the proposed estimator $\hat{d}_h$ is better than the conventional unbiased estimators.

5. Illustrations

Case I—Estimation of Common Parameters of Two Normal Populations

(i) To illustrate above procedure of estimation, we consider the estimator

$$d_4 = \lambda_1 \bar{x}_1 + \lambda_2 \bar{x}_2$$

for estimating θ.

It has been shown in Section 1 that the estimator d_4 has minimum MSE for the optimum values of weights λ_1 and λ_2 are given by

$$\left.\begin{aligned} \lambda_{10} &= a_2/(a_1 + a_2 + a_1 a_2) \\ \text{and} \quad \lambda_{20} &= a_1/(a_1 + a_2 + a_1 a_2) \end{aligned}\right\} \tag{4.5}$$

As stated earlier, using the guessed (or estimated) values of C_1 and C_2, we find the guessed (or estimated) values of λ_{10} and λ_{20}, respectively, which are

$$\left.\begin{aligned}\hat{\lambda}_{10} &= a_2/(a_1 + a_2 + \alpha^2 a_1 a_2)\\ \text{and} \quad \hat{\lambda}_{20} &= a_1/(a_1 + a_2 + \alpha^2 a_1 a_2)\end{aligned}\right\} \tag{4.6}$$

Thus, the estimator d_4 becomes

$$\hat{d}_4 = \hat{\lambda}_{10}\, \bar{x}_1 + \hat{\lambda}_{20}\, \bar{x}_2 \tag{4.7}$$

The bias and MSE of $\hat{d}_4$ are, respectively, given by

$$B(\hat{d}_4) = -\frac{\theta\, \alpha^2 a_1 a_2}{(a_1 + a_2 + \alpha^2 a_1 a_2)} \tag{4.8}$$

and

$$\text{MSE}(\hat{d}_4) = \theta^2 \frac{a_1 a_2 (a_1 + a_2 + \alpha^4 a_1 a_2)}{(a_1 + a_2 + \alpha^2 a_1 a_2)^2} \tag{4.9}$$

The MSE of $\hat{d}_4$ will be smaller than that of conventional unbiased estimator d_1 if the following in equality :

$$\alpha^2 \leqslant \frac{2(a_1 + a_2)}{(a_1 + a_2 - a_1 a_2)} \tag{4.10}$$

holds good. This inequality resembles with that of Pandey and Singh [6]. For given values of C_1, C_2, n_1 and n_2, the ranges of α can be calculated.

(ii) We consider an estimator

$$d_6 = \lambda_1 s_1^2 + \lambda_2 s_2^2 \text{ for } \theta^2. \tag{4.11}$$

For the optimum weights

$$\lambda_{10} = \frac{(n_1 - 1)}{(n_1 + n_2)\, C_1^2} \text{ and } \lambda_{20} = \frac{(n_2 - 1)}{(n_1 + n_2)\, C_2^2} \tag{4.12}$$

the minimum MSE of d_6 is given by

$$\text{MSE}(d_6) = \frac{2\,\theta^4}{(n_1 + n_2)} \tag{4.13}$$

It should be noted that the minimum MSE of d_6 is true only when λ_{10} and λ_{20} are known exactly. But it is not in general true. In such circumstances, we adopt the procedure discussed above. The estimated (or guessed) values of λ_{10} and λ_{20} are, respectively, given by

$$\hat{\lambda}_{10} = \frac{(n_1 - 1)}{(n_1 + n_2)\,\alpha^2\, C_1^2}, \text{ and } \hat{\lambda}_{20} = \frac{(n_2 - 1)}{(n_1 + n_2)\,\alpha^2\, C_2^2} \tag{4.14}$$

Thus, the estimator d_6 turns out to be

$$\hat{d}_6 = \hat{\lambda}_{10}\, s_1^2 + \hat{\lambda}_{20}\, s_2^2 \tag{4.15}$$

The bias and MSE of $\hat{d}_6$ are, respectively given by

$$B(\hat{d}_6) = \theta^2\,[(n_1 + n_2)(1 - \alpha^2) - 2]/[(n_1 + n_2)\,\alpha^2]$$

and

$$\text{MSE}(\hat{d}_6) = \theta^4\,[(n_1 + n_2)(1 - \alpha^2)^2 - 2(1 - 2\alpha^2)]/[(n_1 + n_2)\,\alpha^4], \tag{4.16}$$

The MSE of $\hat{d}_6$ will be lesser than the variance of conventional unbiased estimator d_2 defined in (1.2) if the following inequality

$$\alpha^4 - 2\,\alpha^2\, b^* + b^* \leqslant 0 \tag{4.17}$$

holds true, where $b^* = (n_1 + n_2 - 2)^2/[(n_1 + n_2)(n_1 + n_2 - 4)]$. For given values of n_1 and n_2, one can calculate the ranges of α.

Case II : *Estimation of Common Parameters in Single Normal Population*

(i) We consider the estimator of θ proposed by Govindarajulu and Sahai [3], Gleser and Healy [2] and Upadhyaya and Singh [21]

$$\text{as } e_4 = w_0\, s + w_1\, \bar{x} \tag{4.18}$$

where w_0 and w_1 are suitably chosen weights to be determined such that MSE of e_4 is minimum.

For the optimum values

$$w_{00} = \frac{f_n\, C}{n\,[1 + n^{-1}\, C^2 - f_n^2)} \text{ and } w_{10} = \frac{(1 - f_n^2)}{[1 + n^{-1}\, C^2 - f_n^2]} \tag{4.19}$$

of weights w_0 and w_1, respectively, the minimum MSE of e_4 is given by

$$\text{Min. MSE}(e_4) = \frac{\theta^2}{n} \cdot \frac{C^2\,(1 - f_n^2)}{(1 + n^{-1}\, C^2 - f_n^2)} \tag{4.20}$$

where, $f_n = \sqrt{\left(\frac{2}{n-1}\right)} \left\{\sqrt{\left(\frac{n}{2}\right)} \Big/ \sqrt{\left(\frac{n-1}{n}\right)}\right\}$.

In case weights in (4.19) are unknown then we have to use the guessed (or estimated) values of C as $\hat{c} = \alpha C, (\alpha > 0)$ and hence the weights are

$$\left.\begin{aligned} \hat{w}_{00} &= \frac{f_n \alpha C}{n[1 + n^{-1}\alpha^2 C^2 - f_n^2]} \\ \text{and} \quad \hat{w}_{10} &= \frac{(1 - f_n^2)}{[1 + n^{-1}\alpha^2 C^2 - f_n^2]} \end{aligned}\right\} \tag{4.21}$$

Thus, the estimator e_4 takes the form

$$\hat{e}_4 = \hat{w}_{00}\, s + \hat{w}_{10}\, \bar{x} \tag{4.22}$$

The bias and MSE of $\hat{e}_4$ are given by

$$B(\hat{e}_4) = -\theta \frac{[C^2 \alpha (\alpha - f_n^2) + nf_n (1 - f_n)]}{n[1 + n^{-1}\alpha^2 C^2 - f_n^2]} \quad \text{and}$$

$$\text{MSE}\,(\hat{e}_4) = \frac{\theta^2 C^2}{n^2} \cdot \frac{[\alpha^2 C^2 (\alpha - f_n)^2 + n(1 - f_n^2)^2]}{[1 + n^{-1} C^2 \alpha^2 - f_n^2]^2} \tag{4.23}$$

respectively. The MSE of $\hat{e}_4$ will be smaller than the conventional unbiased estimator if the inequality

$$\alpha^2 - \alpha \left\{\frac{2nf_n^2}{(n-1)C^2}\right\} + \frac{n(3f_n^2 - 2)}{(n-1)C^2} \leqslant 0 \tag{4.24}$$

For the given values of n and C, the ranges of α can be obtained.

(ii) We consider the estimator proposed by Searls [9] as

$$e_1 = w\bar{x}, \text{ where } w \text{ is a constant.} \tag{4.25}$$

For the optimum weight $w_0 = n/(n + C^2)$ (4.26)

the minimum MSE of e_1 is given by Min · MSE $(e_1) = (\theta^2 C^2)(n + C^2)$ (4.27)

If C is unknown, then C is replaced by $\hat{c} = \alpha C, (\alpha > 0)$

and hence, $\hat{w}_0 = n/(n + \alpha^2 C^2)$. (4.28)

Replacing w_0 by $\hat{w}_0$ in (4.25) we get $\hat{e}_1 = \hat{w}_0 \bar{x}$. The bias and MSE of $\hat{e}_1$ are respectively given by

$$B(\hat{e}_1) = - \frac{\alpha^2 C^2 \theta}{(n + \alpha^2 C^2)} \quad \text{and}$$

$$\text{MSE}(\hat{e}_1) = \frac{\theta^2 (n + \alpha^4 C^2) C^2}{(n + \alpha^2 C^2)^2} \tag{4.30}$$

The MSE of $\hat{e}$ will be samaller than that of usual unbiased estimator $\bar{x}$ if

$$\alpha^2 \leqslant \frac{2n}{(n - C^2)} \tag{4.31}$$

which is same as obtained by Hirano [4]. He has also obtained the ranges of α for given values of n and C.

(iii) We consider the estimator suggested by Singh *et al.* [12] as

$$t_7 = w_0 s^2 \tag{4.32}$$

where w_0 is a suitably chosen constant. For the optimum value of w_0,

$$w_{00} = \frac{(n - 1)}{(n + 1) C^2} \tag{4.33}$$

the minimum MSE of t_7 is given by

$$\text{Min} \cdot \text{MSE}(t_7) = \frac{2\theta^4}{n + 1}, \tag{4.34}$$

If C is unknown, then C is replaced by $\hat{c} = \alpha C$, $(\alpha > 0)$ and thus w_{00} becomes

$$\hat{w}_{00} = \frac{(n - 1)}{(n + 1) \alpha^2 C^2} \tag{4.35}$$

Substituting $\hat{w}_{00}$ in place of w_0 in (4.32) we get

$$t_7 = \hat{w}_{00} s^2 \tag{4.36}$$

The bias and MSE of $\hat{t}_7$ are respectively given by

$$B(\hat{t}_7) = - \theta^2 [(n + 1) \alpha^2 - n + 1]/\{(n + 1) \alpha^2\} \quad \text{and}$$

$$\text{MSE}(t_7) = \frac{\theta^4}{(n + 1) \alpha^4} [(n + 1) \alpha^4 + (n - 1)(1 - 2\alpha^2)] \tag{4.37}$$

which will be smaller than that of unbiased estimator $t_9 = s^2/C^2$ if the inequality

$$\left[\alpha^4 - 2\,\alpha^2 \frac{(n-1)^2}{(n+1)\,(n-3)} + \frac{(n-1)^2}{(n+1)\,(n-3)}\right] \leqslant 0 \quad (4.38)$$

holds good. For given values of n, the ranges of α can be calculated.

In similar manner the biases and MSE's of several estimators can be obtained, in the situation where coefficients of variation are not exactly known, and their merits can be examined over conventional unbiased estimators.

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